The Applied Measurement of Mortality: A Critique and a Revision

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Abstract

In countries with adequate mortality statistics, the common applied approach to measuring mortality is based on the establishment of age-specific empirical *annual occurrence / exposure rates* M_x. Here, the number of recorded events of death in an age group and during a given year (the occurrence of events) is divided by a measure of the corresponding age-specific population at risk of dying during that year (the exposure).

In a next step, the resulting rates are converted to probabilities of dying q_x. In the common period approach to life table construction, the complements of these age-specific probabilities of dying, $1 - q_x$, are then applied in sequence to a birth cohort of size ℓ_0. The result is a synthetic cohort ℓ_x describing survivorship from birth under the empirically observed *mortality regime*. Life expectancies e_x are then evaluated in a routine manner.

While long-established standard applied practice and while computationally convenient, scientifically this approach is, however, less than optimal.

In particular, the occurrence / exposure rates M_x themselves are fundamentally flawed on *mathematical* grounds, on *methodological* grounds, and on *empirical* grounds. This paper explains and examines these flaws.

In applied practice, furthermore, it is common to present the results for *synthetic cohorts*, such as survivorship statistics and life expectancies, *as if* they could be taken as indicative of the actual or future empirical experience of real cohorts over time and age. On an aside the paper also notes that, as an assembly of the mortality experience of a sequence of distinct cohorts at one given point in time, such applied use of synthetic cohorts is questionable practice from a scientific point of view.

The paper then continues to discuss how these flaws can be mitigated and even be avoided altogether. Using conventionally available data on deaths and well-established mathematical methods, it discusses and demonstrates how instead fully consistent *annually-stationary* (that

is, constant within a given year) empirical *occurrence / exposure rates* of the type M_x can be obtained as a matter of routine.

However, the paper goes beyond this: Using the same conventional data on deaths and the same mathematical approach, it continues to show how to obtain the more precise and more informative *time-continuous instantaneous mortality rates* or *mortality intensities* $\mu(x)$.

These instantaneous rates $\mu(x)$ are also shown completely and accurately to capture the underlying but in itself unobservable *force of mortality* which is responsible for the actual empirically observed mortality pattern.

Further, all results are shown to be *mathematically consistent* and *methodologically sound*. Also, the results can be seen to be *empirically maximally informative*, that is, they fully capture and characterize actual mortality processes *in complete detail* as time and age progress.

In addition, the approach to measurement presented in this paper is shown to be mathematically and empirically fully consistent along *cohort* lines, altogether avoiding the need to resort to synthetic cohorts.

The paper concludes by highlighting how this new and improved approach to measuring mortality has significant implications for common applied methods of *life table construction* and the evaluation of life expectancies, as well as for established methods of *population projection and forecasting*.